Effective action of (massive) IIA on manifolds with $SU(3)$ structure.

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Abstract

In this paper we consider compactifications of massive type IIA supergravity on manifolds with $SU(3)$ structure. We derive the gravitino mass matrix of the effective four-dimensional $\mathcal{N} = 2$ theory and show that vacuum expectation values of the scalar fields naturally induce spontaneous partial supersymmetry breaking. We go on to derive the superpotential and the Kähler potential for the resulting $\mathcal{N} = 1$ theories. As an example we consider the $SU(3)$ structure manifold $SU(3)/U(1) \times U(1)$ and explicitly find $\mathcal{N} = 1$ supersymmetric minima where all the moduli are stabilised at non-trivial values without the use of non-perturbative effects.

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1 Introduction

It has long been hoped that low-energy compactifications of string- and M-theory will lead to phenomenological predictions which could be tested experimentally. A major obstacle to achieving this goal is the presence of many light scalars, or 'moduli', which are typically flat directions of the four-dimensional theory. These arise as the massless modes of the higher dimensional matter fields and as gauge-independent variations of the metric on the compact space. The precise values taken by the moduli in the vacuum yield various parameters in the four-dimensional model and therefore act as predictions coming from string theory. One of the most pressing concerns regarding string theory compactifications, therefore, is the issue of moduli stabilization.

One of the ways of inducing a non-trivial classical potential for the low energy fields in the four-dimensional effective theory is through the inclusion of non-vanishing field strengths for the ten-dimensional fields with directions purely in the internal manifold. These are referred to as fluxes and have been used extensively in the literature for the purposes discussed above—see [1–4] for some of the earlier work. Such fluxes back-react on the internal geometry and will thus typically deform the internal space away from being Ricci flat. In that case the Calabi-Yau manifolds used so often in compactifications can cease to be a true solution of the theory. For that reason compactifications on Calabi-Yau manifolds with fluxes are restricted to the large volume limit where the fluxes are diluted and their back-reaction may be ignored by treating them perturbatively.

Recently a growing body of literature has been looking at including the back-reaction of the fluxes on the internal manifold and considering manifolds which will be true solutions of the theory. These manifolds will have non-vanishing torsion and so the requirements on the compact space for preserving some supersymmetry in the low energy theory is generalized from having special holonomy to admitting a G-structure [5], which can be classified in terms of its non-vanishing torsion classes.

For M-theory most work is done on G_2-structure manifolds [6–9], although $SU(3)$-structure manifolds have also been considered [10, 11]. In the case of string theory both $SU(3)$- and $SU(2)$-structure manifolds have been considered [12–17]. For a general review of structure manifolds in string and M-theory see [18] and references therein.

From the point of view of phenomenology these manifolds have the advantage that, although they are formally more general than Calabi-Yau manifolds, they typically have a much simpler field content. This can be thought of as the torsion placing restrictions on the possible metric deformations of the manifold. They also have the feature that, since they are not Ricci-flat, the four dimensional background will not be Minkowski but (at least in the case where some supersymmetry is preserved) anti-de Sitter.

This outcome is not desired for cosmological reasons and is the reason that the Ricci-flat Calabi-Yau manifolds were originally more attractive candidates. Recently, this reason has become less relevant in the sense that when the inclusion of fluxes does produce a stable vacuum in a Calabi-Yau compactification, that vacuum will typically be anti-de Sitter anyway. It is therefore no more of a problem to start from an anti-de Sitter cosmology in the first place and, like in the Calabi-Yau case [19], hope that some non-perturbative effects will lift this to a Minkowski or a de Sitter vacuum.

Another phenomenologically important feature of $SU(3)$-structure manifolds is that known solutions on these manifolds to ten-dimensional type IIA and IIB supergravities preserve $\mathcal{N} = 1$ supersymmetry [13–17], rather than $\mathcal{N} = 2$ supersymmetry which is problematic as a low-energy symmetry due *inter alia* to its lack of chiral representations.

In this paper we will consider compactifications of Romans' massive type IIA supergravity on manifolds with $SU(3)$ structure. An important advantage of type IIA theory as opposed to type

IIB is that fluxes alone can generate non-trivial potentials for both complex structure and Kähler moduli, although fully stabilising the moduli has required non-perturbative effects such as instanton corrections [20]. Recently, this has been overcome through the use of orientifolds, where $\mathcal{N} = 1$ AdS solutions with all moduli stabilised have been found [21–23].

As yet, the covariant embedding of the massive IIA theory in the M-theory 'web of dualities' is not known, although it is believed to encode information about the type-IIA string theory in D8-brane backgrounds [24]. The massive supergravity theory is also considerably richer than the massless case, and we will not concern ourselves with α' corrections or other 'stringy' effects that would require the full covariant embedding.

We will show that due to the torsion on the internal manifold there are two types of fluxes that are associated with such compactifications: the usual fluxes associated with non-perturbative sources and fluxes originating from vevs of scalar fields. We will then derive the effective low energy $\mathcal{N} = 2$ theory by reducing the gravitino mass terms to obtain the four-dimensional gravitino mass matrix. From this point we will restrict ourselves to the case where the compact space is in a special class of half-flat manifolds shown to be the most general manifolds compatible with the preservation of $\mathcal{N} = 1$ supersymmetry in four dimensions. We will show that for the second type of fluxes the theory can exhibit spontaneous $\mathcal{N} = 2 \to \mathcal{N} = 1$ partial supersymmetry breaking in the vacuum, and construct the resulting $\mathcal{N} = 1$ effective theory.

To study the vacua of the theory, we will consider a particular compact space, and show that an $\mathcal{N} = 1$ supersymmetric vacuum exists where all the moduli are stabilised. This stabilisation does not involve the introduction of any non-perturbative effects into the superpotential or the use of orientifolds.

In section 2 we summarise the relevant tools used to classify $SU(3)$ structure manifolds and show how the structure can be used to induce a metric on the internal manifold. In section 3 we perform a Kaluza-Klein reduction of massive type IIA supergravity to an $\mathcal{N} = 2$ effective four-dimensional theory by deriving the gravitino mass matrix. In section 4 we will go on to derive the superpotential and Kähler potential of the resulting $\mathcal{N} = 1$ effective theory and show how the $\mathcal{N} = 2$ multiplets break to $\mathcal{N} = 1$ superfields. In section 5 we will go through an explicit example of such a compactification on the $SU(3)$-structure manifold $SU(3)/U(1) \times U(1)$. We will derive the effective theory for compactification on this coset and find an explicit supersymmetric minimum where all the fields are stabilised at non-trivial vacuum expectation values. We summarise our conclusions in section 6.

2   Manifolds of $SU(3)$ structure

A six-dimensional manifold is said to have $SU(3)$ structure if it admits a nowhere-vanishing two-form J and three-form Ω (with complex conjugate $\overline{\Omega}$) obeying conditions that we will outline in this section. These forms are typically defined in terms of bilinears of the Killing spinor η_+, which is a nowhere-vanishing Weyl spinor of positive chirality with charge conjugate η_-. Throughout this work, we use the spinor conventions of [16] and so the spinors are normalised to

$$\overline{\eta_+}\eta_+ = \overline{\eta_-}\eta_- = 1 \qquad \overline{\eta_+}\eta_- = \overline{\eta_-}\eta_+ = 0 \ . \tag{2.1}$$

We can then write the structure forms as

$$\begin{aligned} J_{mn} &:= -i\overline{\eta_+}\gamma_{mn}\eta_+ \\ \Omega_{mnp} &:= \overline{\eta_-}\gamma_{mnp}\eta \ , \end{aligned} \tag{2.2}$$

where $\gamma^{m_1 \ldots m_n}$ denote anti-symmetric products of gamma matrices. Note that although (2.2) makes use of the Killing spinor, this spinor does not in fact contain the full information of the structure forms, since the vielbein and hence the metric are implicitly involved in their definition.

We shall now go on to consider the algebraic and differential relations that the structure forms obey, as well as their relation to the metric.

2.1 Algebraic relations

Having defined the $SU(3)$ structure in terms of spinors and gamma matrices, we can then use Fierz rearrangement formulae together with commutation and anticommutation relations for gamma matrices to derive algebraic relations such as

$$
\begin{aligned}
J_m{}^p J_p{}^n &= -\delta_m^n \\
J_m{}^n \Omega_{npq} &= i\Omega_{mpq} \\
(P_+)_m{}^n \Omega_{npq} &= \Omega_{mpq} \\
(P_-)_m{}^n \Omega_{npq} &= 0 \\
\Omega \wedge \overline{\Omega} &= -\frac{4}{3} i J \wedge J \wedge J \\
\Omega \wedge J &= 0 \\
\star\Omega &= -i\Omega \,,
\end{aligned}
\tag{2.3}
$$

where $\star$ denotes the Hodge star and we have defined the usual projectors

$$
(P_\pm)_m{}^n := \frac{1}{2}(\delta_m^n \mp i J_m{}^n) \,.
\tag{2.4}
$$

We note that, in general, manifolds with $SU(3)$ structure are not necessarily Kähler or even complex, despite the existence of a globally defined almost complex structure J. This means that the usual distinction between holomorphic and anti-holomorphic indices will no longer hold globally, however any local results obtained using this distinction will still hold.

2.2 Differential relations and torsion classes

$SU(3)$ structure also implies differential relations between the structure forms J and Ω. To derive these we first examine how the deformation away from $SU(3)$ holonomy is parameterised. The contorsion κ is defined, via the unique connection Γ_T that leaves the structure invariant, using the relation

$$
\kappa_{mn}{}^r := (\Gamma_T)_{[mn]}^r \,.
\tag{2.5}
$$

Γ_T is defined so that the derivatives given by it obey

$$
\nabla_T J = \nabla_T \Omega = 0 \qquad D_T \eta = 0 \,,
\tag{2.6}
$$

where throughout we use ∇ for a space-time covariant derivative and D for a spinor covariant derivative. Making use of this relation, the Levi-Civita derivatives on the structure and spinor are related to the contorsion via

$$
\begin{aligned}
(dJ)_{mnp} &= -6\kappa_{[mn}{}^r J_{p]r} \\
(d\Omega)_{mnpq} &= 12\kappa_{[mn}{}^r \Omega_{pq]r} \\
D_m \eta &= \frac{1}{4}\kappa_{mnp}\gamma^{np}\eta \,.
\end{aligned}
\tag{2.7}
$$

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Note that since the contorsion is antisymmetric in its lowered indices, it is still metric-compatible, so

$$\nabla_T g_{mn} = \nabla g_{mn} = 0 \ . \tag{2.8}$$

The contorsion provides a way of classifying supersymmetric solutions of supergravity theories by considering the structure group G of the manifold as a subgroup of $SO(N)$. Since in general the contorsion has two antisymmetric indices and one other index, we have that

$$\begin{aligned}
\kappa \in \Lambda^1 \otimes \Lambda^2 &\cong& \Lambda^1 \otimes so(n) \\
&\cong& \Lambda^1 \otimes (g \oplus g^\perp) \ ,
\end{aligned} \tag{2.9}$$

where g is the Lie algebra on G and $g^\perp$ is its complement in $so(N)$. Since we know that the action of g on the G-structure must vanish by construction, we can decompose κ according to the irreducible representations of G in $\Lambda^1 \otimes g^\perp$. In the case of $SU(3) \subset SO(6)$, this gives

$$\begin{aligned}
\kappa \in \Lambda^1 \otimes su(3)^\perp &=& (\mathbf{3} + \overline{\mathbf{3}}) \otimes (\mathbf{1} + \mathbf{3} + \overline{\mathbf{3}}) \\
&=& (\mathbf{1} + \mathbf{1}) + (\mathbf{8} + \mathbf{8}) + (\mathbf{6} + \overline{\mathbf{6}}) + (\mathbf{3} + \overline{\mathbf{3}}) + (\mathbf{3} + \overline{\mathbf{3}}) \ .
\end{aligned} \tag{2.10}$$

We then associate each of these bracketed terms with a torsion class $\mathcal{W}$, which in concrete terms means that

$$\begin{aligned}
dJ &=& -\frac{3}{2}\mathrm{Im}(\mathcal{W}_1\overline{\Omega}) + \mathcal{W}_4 \wedge J + \mathcal{W}_3 \\
d\Omega &=& \mathcal{W}_1 J \wedge J + \mathcal{W}_2 \wedge J + \overline{\mathcal{W}_5} \wedge \Omega \ .
\end{aligned} \tag{2.11}$$

The torsion classes can then be used to classify the structure manifold. In particular, for a manifold to be complex we need $\mathcal{W}_1 = \mathcal{W}_2 = 0$, and where $\mathrm{Re}(\mathcal{W}_1) = \mathrm{Re}(\mathcal{W}_2) = \mathcal{W}_4 = \mathcal{W}_5 = 0$ the manifold is *half-flat*. The manifold that we shall go on to consider will be half-flat but not complex. To see why such spaces are not complex, we expand on our final comments in section 2.1, noting that for non-vanishing $\mathcal{W}_1$, the relations in (2.11) simply cannot be written in holomorphic and anti-holomorphic coordinates.

A Calabi-Yau manifold thus has an alternative definition as a manifold of $SU(3)$ structure with completely vanishing torsion classes. Although in this sense, considering $SU(3)$ structure manifolds with non-trivial torsion is more general than the Calabi-Yau case, in fact the physics that we obtain from such manifolds will often be simpler. For example, it was argued in [25] that nearly-Kähler manifolds do not possess any complex structure moduli, and this argument should also apply to the half-flat manifolds that we will consider.

2.3 Induced metric

Having an $SU(3)$ structure on a manifold is a stronger condition then having a metric. In fact the forms J and Ω induce a metric on the space via the relation

$$\begin{aligned}
g_{mn} &=& s^{-1/8} s_{mn} \ \text{for} \\
s_{mn} &=& -\frac{1}{64}(\Omega_{mpq}\overline{\Omega}_{nrs} + \Omega_{npq}\overline{\Omega}_{mrs})J_{tu}\hat{\epsilon}^{pqrstu} \ ,
\end{aligned} \tag{2.12}$$

where s is the determinant of s_{mn}. This form for the metric allows us to express variations of the metric in terms of variations of the $SU(3)$-structure

$$\begin{aligned}
\delta g_{mn} &=& -\frac{1}{8}(\delta\Omega)_{(m}{}^{pq}\overline{\Omega}_{n)pq} - \frac{1}{8}(\delta\overline{\Omega})_{(m}{}^{pq}\Omega_{n)pq} - (\delta J)_{t(m}J^t{}_{n)} \\
&& + \left[\frac{1}{64}(\delta\Omega)\lrcorner\overline{\Omega} + \frac{1}{64}(\delta\overline{\Omega})\lrcorner\Omega - \frac{1}{8}(\delta J)\lrcorner J\right] g_{mn} \ .
\end{aligned} \tag{2.13}$$

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In this form, calculation is rather difficult, however by using the fact that $P_+ + P_- = \mathbb{1}$, we can obtain some of the calculational convenience of holomorphic and anti-holomorphic coordinates by acting on (2.13) with projectors to give

$$
\begin{aligned}
(P_+)_m{}^p (P_+)_n{}^q \delta g_{pq} &= -\frac{1}{8} \delta \overline{\Omega}_p{}^{qr} (P_+)_{(m}{}^p \Omega_{n)qr} \\
(P_-)_m{}^p (P_-)_n{}^q \delta g_{pq} &= -\frac{1}{8} \delta \Omega_p{}^{qr} (P_-)_{(m}{}^p \overline{\Omega}_{n)qr} \\
\left[(P_+)_m{}^p (P_-)_n{}^q + (P_-)_m{}^p (P_+)_n{}^q \right] \delta g_{pq} &= -\frac{1}{8} \delta \Omega_p{}^{qr} (P_+)_{(m}{}^p \overline{\Omega}_{n)qr} - \frac{1}{8} \delta \overline{\Omega}_p{}^{qr} (P_-)_{(m}{}^p \Omega_{n)qr} - \delta J_{p(m} J_{n)}{}^p \\
&\quad + \left[\frac{1}{64} (\delta \Omega) \lrcorner \overline{\Omega} + \frac{1}{64} (\delta \overline{\Omega}) \lrcorner \Omega - \frac{1}{8} (\delta J) \lrcorner J \right] g_{mn} . \quad (2.14)
\end{aligned}
$$

Variations of the metric can, therefore, still be encoded in terms of variations of J and Ω, which we will refer to as Kähler and complex structure deformations respectively.

3 Reduction of the IIA action

In this section we will consider reducing the ten-dimensional action for massive type IIA supergravity on a general manifold with $SU(3)$ structure. We will begin by summarising Romans' massive type IIA supergravity. We will then show how to decompose the ten-dimensional metric, Ricci scalar, dilaton, form fields and gravitino. Reducing the terms that give gravitino mass terms will lead to an effective $\mathcal{N} = 2$ theory, which will be specified by the four dimensional gravitino mass matrix.

3.1 Action and field content

The action for massive type IIA supergravity, first outlined in [26], in the Einstein frame reads

$$
\begin{aligned}
S_{IIA}^{10} &= \int \left(\frac{1}{2} \hat{R} \star 1 - \frac{1}{4} d\hat{\phi} \wedge \star d\hat{\phi} - \frac{1}{4} e^{-\hat{\phi}} \hat{F}_3 \wedge \star \hat{F}_3 - \frac{1}{4} e^{\frac{1}{2}\hat{\phi}} \hat{F}_4 \wedge \star \hat{F}_4 \right. \\
&\quad - m^2 e^{\frac{3}{2}\hat{\phi}} \hat{B}_2 \wedge \star \hat{B}_2 - m^2 e^{\frac{5}{2}\hat{\phi}} \star 1 \\
&\quad \left. + \frac{1}{4} d\hat{C}_3 \wedge d\hat{C}_3 \wedge \hat{B}_2 + \frac{1}{6} m d\hat{C}_3 \wedge \hat{B}_2 \wedge \hat{B}_2 \wedge \hat{B}_2 + \frac{1}{20} m^2 \hat{B}_2 \wedge \hat{B}_2 \wedge \hat{B}_2 \wedge \hat{B}_2 \wedge \hat{B}_2 \right) \\
&\quad + \int \sqrt{-\hat{g}} d^{10} X \left[- \overline{\hat{\Psi}}_M \Gamma^{MNP} D_N \hat{\Psi}_P - \frac{1}{2} \overline{\hat{\lambda}} \Gamma^M D_M \hat{\lambda} - \frac{1}{2} (d\hat{\phi})_N \overline{\hat{\lambda}} \Gamma^M \Gamma^N \hat{\Psi}_M \right. \\
&\quad - \frac{1}{96} e^{\frac{1}{4}\hat{\phi}} (\hat{F}_4)_{PRST} \left(\overline{\hat{\Psi}}^M \Gamma_{[M} \Gamma^{PRST} \Gamma_{N]} \hat{\Psi}^N + \frac{1}{2} \overline{\hat{\lambda}} \Gamma^M \Gamma^{PRST} \hat{\Psi}_M + \frac{3}{8} \overline{\hat{\lambda}} \Gamma^{PRST} \hat{\lambda} \right) \\
&\quad + \frac{1}{24} e^{-\frac{1}{2}\hat{\phi}} (\hat{F}_3)_{PRS} \left(\overline{\hat{\Psi}}^M \Gamma_{[M} \Gamma^{PRS} \Gamma_{N]} \Gamma_{11} \hat{\Psi}^N + \overline{\hat{\lambda}} \Gamma^M \Gamma^{PRS} \Gamma_{11} \hat{\Psi}_M \right) \\
&\quad + \frac{1}{4} m e^{\frac{3}{4}\hat{\phi}} \hat{B}_{PR} \left(\overline{\hat{\Psi}}^M \Gamma_{[M} \Gamma^{PR} \Gamma_{N]} \Gamma_{11} \hat{\Psi}^N + \frac{3}{4} \overline{\hat{\lambda}} \Gamma^M \Gamma^{PR} \Gamma_{11} \hat{\Psi}_M + \frac{5}{8} \overline{\hat{\lambda}} \Gamma^{PR} \Gamma_{11} \hat{\lambda} \right) \\
&\quad \left. - \frac{1}{2} m e^{\frac{5}{4}\hat{\phi}} \overline{\hat{\Psi}}_M \Gamma^{MN} \hat{\Psi}_N - \frac{5}{4} m e^{\frac{5}{4}\hat{\phi}} \overline{\hat{\lambda}} \Gamma^M \hat{\Psi}_M + \frac{21}{16} m e^{\frac{5}{4}\hat{\phi}} \overline{\hat{\lambda}} \hat{\lambda} \right] . \quad (3.1)
\end{aligned}
$$

This action is a generalisation of the type IIA supergravity that is obtained from the low-energy limit of type IIA string theory, although some care must be taken when taking the massless limit $m \to 0$ [26].

We now turn to notation and field content. The indices $M, N \ldots$ run from 0 to 9, and the ten dimensional space-time coordinates are X^M. In the Neveau-Schwarz-Neveau-Schwarz (NS-NS) sector the action contains the bosonic fields $\hat{\phi}, \hat{B}_2, \hat{g}$, which are the ten-dimensional dilaton, a massive two-form and the metric, together with the fermionic fields $\hat{\Psi}, \hat{\lambda}$, which are the gravitino and dilatino. The Ramond-Ramond (RR) sector contains the three-form $\hat{C}_3$ and a one-form $\hat{A}^0$ which is eliminated by a gauge transformation of $\hat{B}_2$ as in [26]. The field strengths in the action are given by

$$\begin{aligned}
\hat{F}_4 &:= d\hat{C}_3 + m\hat{B}_2 \wedge \hat{B}_2 & (3.2) \\
\hat{F}_3 &:= d\hat{B}_2 . & (3.3)
\end{aligned}$$

Note that, in contrast to the massless case, $\hat{F}_4$ will not in general be closed, and that due to the equations of motion neither field strength will in general be co-closed.

3.2 Decomposing the metric

We now consider reducing the ten dimensional action on a manifold endowed with $SU(3)$ structure. We split the ten dimensional space-time coordinates as $(X^M) = (x^\mu, y^n)$ with external indices $\mu, \nu \ldots = 0, 1, 2, 3$ and internal indices $m, n \ldots = 4 \ldots 9$. Reflecting the fact that we want the internal space to be compact with compactification radii significantly smaller than any length scales we wish to consider in four dimensions, we decompose the ten-dimensional metric into a sum of four-dimensional and six-dimensional metrics

$$\hat{g}_{MN}(X) dX^M dX^N = \Delta(y) g_{\mu\nu}(x) dx^\mu dx^\nu + g_{mn}(x, y) dy^m dy^n. \qquad (3.4)$$

$\Delta(y)$ is a possible warp factor which will give the four dimensional metric dependence on the internal coordinates. In section (4.1) we will discuss the most general solution of massive type IIA supergravity on manifolds with $SU(3)$ structure [16] that preserves $\mathcal{N} = 1$ supersymmetry, where it was shown that $\Delta(y)$ is in fact constant. We therefore consistently set it to unity. We note, however, that in the case where supersymmetry is completely broken this warp factor may be non vanishing. (3.4) also determines how the Dirac matrices decompose and so we have

$$\Gamma^\mu := \gamma^\mu \otimes \gamma_7 \qquad \Gamma^m := \gamma_5 \otimes \gamma^m , \qquad (3.5)$$

where $\{\gamma^\mu\}, \{\gamma^m\}$ furnish representations of the four- and six-dimensional Dirac matrices respectively.

3.3 Ricci scalar reduction

Given the choice of metric ansatz above, the ten-dimensional Ricci scalar can be written as

$$\hat{R} = R + R_6 - g^{mn} \nabla^2 g_{mn} - \frac{1}{4} g^{mn} g^{pq} \left(\partial g_{mn} \cdot \partial g_{pq} - 3 \partial g_{mp} \cdot \partial g_{nq} \right) , \qquad (3.6)$$

where R, R_6 are the four- and six-dimensional Ricci scalars respectively. ∂, ∇ are four-dimensional derivatives, with $\cdot$ representing contraction over four-dimensional indices. We shall reduce both the Einstein-Hilbert and dilaton kinetic terms at the same time, which are given from (3.1) as

$$S_{EH,D}^{10} = \int d^{10}x \sqrt{-g} \left(\frac{1}{2} \hat{R} - \frac{1}{4} \partial_M \hat{\phi} \partial^M \hat{\phi} \right) . \qquad (3.7)$$

Following the reduction of these terms, there are three field redefinitions for the effective four-dimensional action that are needed to put the kinetic terms in canonical form

$$g_{\mu\nu} \quad \to \quad \mathcal{V}^{-1} g_{\mu\nu} \tag{3.8}$$

$$g_{mn} \quad \to \quad e^{-\hat{\phi}/2} g_{mn} \tag{3.9}$$

$$\phi \quad := \quad \hat{\phi} - \frac{1}{2}\ln\mathcal{V} \,, \tag{3.10}$$

where ϕ is the four-dimensional dilaton. This gives a final form for the four-dimensional action of

$$S_{EH,D}^4 = \int d^4x \sqrt{-g} \left(\frac{1}{2}R + \frac{1}{2}e^{3\phi/2}\mathcal{V}^{-1/4}R_6 - \partial\phi \cdot \partial\phi + \frac{1}{8\mathcal{V}} \int d^6x \sqrt{g}\partial g_{mn} \cdot \partial g^{mn} \right) \,. \tag{3.11}$$

Our task is then to evaluate the internal integral in terms of the $SU(3)$-structure forms, which is possible via the induced metric, as discussed in section 2.3. This allows us to write

$$\frac{1}{2}\partial g_{mn} \cdot \partial g^{mn} = -\partial J_{mn} \cdot \partial J^{mn} + \frac{1}{8}\partial\Omega_{mnp} \cdot \partial\overline{\Omega}^{mnp} \,, \tag{3.12}$$

which we will use later in finding the Kähler potential.

3.4 The Kaluza-Klein expansion forms

It was suggested in [27], and later developed in [28, 29] that a suitable basis for Kaluza-Klein reduction on manifolds of $SU(3)$ structure is given by two-forms ω_i, three-forms α_A, β^A and four-forms $\widetilde{\omega}^i$ obeying the algebraic relations

$$\int \omega_i \wedge \widetilde{\omega}^j = \delta_i^j \qquad \int \alpha_A \wedge \beta^B = \delta_A^B$$

$$\int \alpha_A \wedge \alpha_B = \int \beta^A \wedge \beta^B = 0 \tag{3.13}$$

and the differential relations

$$\begin{aligned}
d\omega_i &= E_{iA}\beta^A - F_i^A\alpha_A \\
d\alpha^A &= E_{iA}\widetilde{\omega}^i \\
d\beta_A &= F_i^A\widetilde{\omega}^i \\
d\widetilde{\omega}^i &= 0 \,,
\end{aligned} \tag{3.14}$$

where the matrices E_{iA} and F_i^A are constant. In the limit where $E_{iA}, F_i^A \to 0$, we recover the usual set of harmonic forms for a Calabi-Yau compactification: $\{\omega_i, \widetilde{\omega}^j\}_{i,j=1}^{h^{1,1}}, \alpha_0, \beta^0, \{\alpha_a, \beta^b\}_{a,b=1}^{h^{2,1}}$, where the $h^{p,q}$s are the Hodge numbers of the manifold. For the case where $E_{iA}, F_i^A \neq 0$, however, it has been shown in [29] that the relevant forms do not carry topological information, and so there is no metric-independent interpretation of the expansion forms.

Forms satisfying (3.13) and (3.14) were shown to be the correct basis for the case of half-flat manifolds with Calabi-Yau mirror manifolds. It is natural to extend their use to general half-flat manifolds, and it has been conjectured that such forms could in fact be applied to general $SU(3)$-structure compactifications [29]. With this understood, we shall proceed to make use of them whilst bearing in mind that other bases for Kaluza-Klein reduction are not mathematically excluded.

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3.5 Decomposing the form fields and fluxes

We decompose the ten-dimensional form fields in the following way:

$$\hat{B}_2(X) = B(x) + \mathring{B}(y) + b(x, y) \tag{3.15}$$

$$\hat{C}_3(X) = C(x) + \mathring{C}(y) + c(x, y) \ . \tag{3.16}$$

Here B and C are external two and three-forms respectively. $\mathring{B}$ and $\mathring{C}$ are internal two and three-forms with no dependence on external co-ordinates. They give rise to NS-NS and RR flux respectively. b and c are two and three-forms that depend on both the internal and external manifolds. Using the basis (3.13) we can expand them as

$$
\begin{aligned}
b(x, y) &= b^i(x)\omega_i(y) \tag{3.17}\\
c(x, y) &= \xi^A(x)\alpha_A(y) - \widetilde{\xi}_A(x)\beta^A(y) + A^i(x) \wedge \omega_i(y) \ , \tag{3.18}
\end{aligned}
$$

where A^i are space-time vectors. Given our decomposition of the form fields, the field strengths introduced in (3.3) can be written as

$$
\begin{aligned}
\hat{F}_4 &:= d\hat{C}_3 + m\hat{B}_2 \wedge \hat{B}_2 \\
&= d_4(C + c) + d_6(\mathring{C} + c) + m(B + \mathring{B} + b) \wedge (B + \mathring{B} + b) \\
\hat{F}_3 &:= d\hat{B}_2 \\
&= d_4(B + b) + d_6(\mathring{B} + b) \ , \tag{3.19}
\end{aligned}
$$

where d_4 and d_6 denote exterior derivatives on the external and internal spaces respectively. We shall usually suppress these subscripts. Of particular interest are the internal parts of the field strengths (fluxes) which are given by

$$
\begin{aligned}
F_3 &:= d(\mathring{B} + b) =: dB_2 \tag{3.20}\\
F_4 &:= d(\mathring{C} + c) + m(\mathring{B} + b) \wedge (\mathring{B} + b) \ . \tag{3.21}
\end{aligned}
$$

In contrast to the usual situation for flux compactifications where fluxes obtain their values entirely from the 'background' field strengths $\mathring{B}$ and $\mathring{C}$ (whose precise form is typically not known) the fluxes here can receive contributions from the vacuum expectation values (vevs) of the fields b and c.[1] This difference comes partly because the exterior derivative does not automatically vanish on these fields and partly because the flux $\hat{F}_4$ has a non-exact contribution from the second term in (3.21). To distinguish between those fluxes that arise in the traditional way and those that arise from vevs, we further define

$$H_3 := d\mathring{B} \qquad G_4 := d\mathring{C} + m\mathring{B} \wedge \mathring{B} \ . \tag{3.22}$$

Now, to preserve Poincaré invariance of the four-dimensional theory, all external components of the field strengths must be proportional to the four-dimensional volume form. This restricts us to the only allowed external field strength of

$$(\hat{F}_4)_{\mu\nu\rho\sigma} = f\epsilon_{\mu\nu\rho\sigma} \ , \tag{3.23}$$

---

[1] We note here that, as can be seen from (3.20) and (3.21), the splitting of the two types of contribution to the flux is arbitrary. We could have defined $\mathring{B}$ and $\mathring{C}$ to include the vevs of the scalars and then the scalars would have zero vevs by definition. We have, however, chosen to keep the distinction between the two types more apparent by considering $\mathring{B}$ and $\mathring{C}$ as arising from sources other than the scalar vevs.

where, due to its similarity with a similar parameter in the eleven-dimensional case, we will call f a Freud-Rubin parameter. The Freud-Rubin parameter can be calculated in terms of the matter fields by considering the dualisation of the external three-form $C(x)$. Reducing the relevant terms in (3.1) gives the four-dimensional action for $C(x)$

$$S_C^{(4)} = \int_{X_4} \left[-\frac{1}{4} \mathcal{V} e^{\frac{1}{2}\hat{\phi}} (dC + mB \wedge B) \wedge \star (dC + mB \wedge B) + \frac{1}{2} AdC \right] , \qquad (3.24)$$

where

$$A := \int_Y \left[d\mathring{C} \wedge \mathring{B} + b \wedge d\mathring{C} + dc \wedge \mathring{B} + \frac{1}{3} m\mathring{B} \wedge \mathring{B} \wedge \mathring{B} + m\mathring{B} \wedge \mathring{B} \wedge b + m\mathring{B} \wedge b \wedge b + \frac{1}{3} mb \wedge b \wedge b \right] . \qquad (3.25)$$

To dualise C we follow the discussion in [30] and add a Lagrange multiplier λ

$$S_C^{(4)} = \int_X \left[-\frac{1}{4} \mathcal{V} e^{\frac{1}{2}\hat{\phi}} (dC + mB \wedge B) \wedge \star (dC + mB \wedge B) + \frac{1}{2} AdC + \frac{1}{2} \lambda dC \right] . \qquad (3.26)$$

Taking the equation of motion for C and substituting in (3.23) then gives

$$\star (dC + mB \wedge B) = \mathcal{V}^{-1} e^{-\frac{1}{2}\hat{\phi}} (A + \lambda) = -f , \qquad (3.27)$$

which allows us to write f in terms of the four-dimensional constant λ and the integral (3.25). We note here that we do not need to perform a similar dualisation for $\hat{B}_2$, since although a massless two-form would have been dual to a scalar, a massive two-form will be dual to a massive vector, which we are not considering in our analysis.

3.6 The geometrical moduli

We now turn to the fields arising from metric deformations. From general $\mathcal{N} = 2$ supergravity considerations [31, 32], we know that the complex structure deformations z^a span a special Kähler manifold $\mathcal{M}^{cs}$ with a unique holomorphic three-form Ω^{cs}, which has *periods* Z^A and $F_A(Z^A)$, that are homogeneous functions of the z^as. The Kähler potential is then given by the symplectic inner product

$$K^{cs} := -\ln i \left\langle \Omega^{cs} | \overline{\Omega}^{cs} \right\rangle = -\ln i \left[\bar{Z}^A F_A - Z^A \bar{F}_A \right] =: -\ln(||\Omega^{cs}||^2 \mathcal{V}) . \qquad (3.28)$$

We can then use the forms in section 3.4 to expand Ω^{cs} and re-write the Kähler potential as below

$$\Omega^{cs} = Z^A \alpha_A - F_A \beta^A \qquad (3.29)$$

$$K^{cs} = -\ln i \int \Omega^{cs} \wedge \overline{\Omega}^{cs} . \qquad (3.30)$$

We are now interested in relating Ω^{cs} to the holomorphic three-form Ω in (2.2). Writing

$$\Omega^{cs} = \frac{1}{\sqrt{8}} ||\Omega^{cs}|| \Omega , \qquad (3.31)$$

we see that inserting (3.31) into (3.30) and using (2.3) we recover (3.28). As a check on this process, we note that inserting the relation (3.31) into (2.14) and going to a local patch where we can write global holomorphic and anti-holomorphic coordinates, the usual relations for metric variations are obtained

$$\delta g_{\alpha\overline{\beta}} = -i\delta J_{\alpha\overline{\beta}} , \qquad \delta g_{\alpha\beta} = -\frac{1}{||\Omega^{cs}||^2} \left(\delta\overline{\Omega}^{cs} \right)_\alpha{}^{\gamma\delta} (\Omega^{cs})_{\beta\gamma\delta} . \qquad (3.32)$$

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$g_{\mu\nu}, A^0$	gravitational multiplet
$\xi^0, \tilde{\xi}_0, \phi, B$	tensor multiplet
b^i, v^i, A^i	vector multiplets
$\xi^a, \tilde{\xi}_a, z^a$	hypermultiplets

Table 1: Table showing the $\mathcal{N} = 2$ multiplets in type IIA theory

The Kähler structure deformations v^i arise in the usual way, after we expand J in the forms from section 3.4, which gives

$$J = v^i \omega_i \tag{3.33}$$

$$K = -\ln \frac{4}{3} J \wedge J \wedge J . \tag{3.34}$$

Inserting (3.17) into (3.1) and (3.33) into (3.11) we see that the Kähler structure deformations v^i combine with the NS-NS scalars b^i to span a special Kähler manifold $\mathcal{M}^{SK}$ with Kähler potential (3.34).

In summary, the geometrical moduli fields combine with the massless modes of the matter fields to form $\mathcal{N} = 2$ multiplets as shown in Table 1. The hypermultiplets span a quaternionic manifold $\mathcal{M}^Q$ with a special Kähler submanifold $\mathcal{M}^{cs}$ and the vector multiplets span the special Kähler manifold $\mathcal{M}^K$.

3.7 Decomposing the gravitino

Before we write down the mass matrix for the gravitini, we have to choose an appropriate ansatz for the ten-dimensional gravitino. As discussed in section 2 the internal manifold, which has $SU(3)$ structure, supports a single globally defined, positive-chirality Weyl spinor η_+ and its charge conjugate η_-, which will have negative chirality. From standard arguments, we expect terms involving other spinors on the internal space to lead to four-dimensional masses at the Kaluza-Klein scale, and so they can be ignored. Given $\mathcal{N} = 2$ supersymmetry, we further expect the external degrees of freedom for the gravitino to be given by a single Dirac spinor which can be decomposed as two independent Weyl spinors. The most general spinor ansatz for the ten dimensional gravitino that involves these degrees of freedom is then

$$\hat{\Psi}_M = \psi_{M\alpha} \otimes (a^\alpha \eta_+ + b^\alpha \eta_-) + \psi_M^\alpha \otimes (c_\alpha \eta_+ + d_\alpha \eta_-) , \tag{3.35}$$

where the indices α, β are $SU(2)$ indices, which imply positive chirality of a spinor when lowered and negative chirality when raised. $a^\alpha, b^\alpha, c_\alpha, d_\alpha$ are complex numbers. $\psi_{\mu 1,2}$ are thus four-dimensional gravitini with positive chirality and charge conjugates $\psi_\mu^{1,2}$, while $\psi_{m1,2}$ are four-dimensional spin-1/2 fields with charge conjugates $\psi_m^{1,2}$. Note that in order not have cross terms between the gravitini and the spin-1/2 fields the gravitini need to be redefined with some combination of the spin-1/2 fields. This does not affect the mass of the gravitini, however, and so will not be considered here.

There are two physical constraints that we impose on the ansatz (3.35) to restrict it. The first of these is that the ten-dimensional gravitino should be Majorana. This gives the conditions

$$c_{1,2} = -(b^{1,2})^* \qquad d_{1,2} = -(a^{1,2})^* . \tag{3.36}$$

The second constraint is that the gravitino ansatz should yield canonical kinetic terms when reduced, which in this case look like

$$S_{\text{k.t.}}^4 = -\int \sqrt{-g} d^4x \left(\overline{\psi_\mu^1} \gamma^{\mu\rho\nu} D_\rho \psi_{\nu 2} + \overline{\psi_\mu^2} \gamma^{\mu\rho\nu} D_\rho \psi_{\nu 2} \right) + \text{c.c.} , \tag{3.37}$$

where c.c. stands for charge conjugate. The kinetic term for the ten-dimensional gravitino reads

$$S_{\text{k.t.}}^{10} = \int \sqrt{-\hat{g}} d^{10}X \left[-\hat{\overline{\Psi}}_M \Gamma^{MNP} D_N \hat{\Psi}_P \right] . \tag{3.38}$$

Substituting (3.35) into (3.38) and performing the Weyl rescaling (3.8) we get the result that the four-dimensional gravitino kinetic terms will only take the correct form when

$$(a^\alpha)^*(a^\beta) + (b^\alpha)^*(b^\beta) = \frac{1}{2}\mathcal{V}^{-1/2}\delta^{\alpha\beta} . \tag{3.39}$$

Imposing (3.36) and (3.39), together with the absorption of a constant phase into one of the spinor degrees of freedom, gives the following form for the gravitino ansatz

$$\hat{\Psi}_M = \frac{1}{2}\mathcal{V}^{-1/4} \left[\quad \psi_{M1} \otimes \left(\sqrt{1/2 + \varepsilon}\, \eta_+ + \sqrt{1/2 - \varepsilon}\, e^{i\theta}\eta_- \right) \right.$$
$$\left. + \quad \psi_{M2} \otimes \left(\sqrt{1/2 - \varepsilon}\, \eta_+ - \sqrt{1/2 + \varepsilon}\, e^{i\theta}\eta_- \right) \right] + \text{c.c.} \tag{3.40}$$

ε can be chosen at convenience by making a further spinor redefinition, while θ is a phase that is not fixed by physical considerations and cannot be absorbed into a spinor redefinition.

Rather than leave these remaining parameters in, we note that upon performing the reduction of terms that give a gravitino mass, it is most convenient to choose $\varepsilon = 0$, while θ can be eliminated by making the redefinitions below, which will not affect the four-dimensional physics

$$\Omega \to e^{i\theta}\Omega \qquad M_{3/2} \to e^{i\theta}M_{3/2} , \tag{3.41}$$

where $M_{3/2}$ is a gravitino mass. This gives us the working ansatz for the gravitino

$$\hat{\Psi}_M = \frac{1}{2\sqrt{2}}\mathcal{V}^{-1/4} \left[\psi_{M1} \otimes (\eta_+ + \eta_-) + \psi_{M2} \otimes (\eta_+ - \eta_-) \right] + \text{c.c.} \tag{3.42}$$

3.8 Gravitino mass matrix

We are interested in the gravitino mass matrix of the $\mathcal{N} = 2$ four-dimensional theory. The terms in the ten-dimensional action (3.1) which will contribute to the gravitino masses are

$$S_{\text{mass}}^{10} = \int \sqrt{-\hat{g}} d^{10}X \left[\quad - \quad \hat{\overline{\Psi}}_\mu \Gamma^{\mu n\nu} D_n \hat{\Psi}_\nu \right.$$
$$- \quad \frac{1}{96}e^{\frac{1}{4}\hat{\phi}}(\hat{F}_4)_{prst}\hat{\overline{\Psi}}^\mu \Gamma_{[\mu}\Gamma^{prst}\Gamma_{\nu]}\hat{\Psi}^\nu$$
$$- \quad \frac{1}{96}e^{\frac{1}{4}\hat{\phi}}(\hat{F}_4)_{\rho\sigma\delta\epsilon}\hat{\overline{\Psi}}^\mu \Gamma_{[\mu}\Gamma^{\rho\sigma\delta\epsilon}\Gamma_{\nu]}\hat{\Psi}^\nu$$
$$+ \quad \frac{1}{24}e^{-\frac{1}{2}\hat{\phi}}(\hat{F}_3)_{prs}\hat{\overline{\Psi}}^\mu \Gamma_{[\mu}\Gamma^{prs}\Gamma_{\nu]}\Gamma_{11}\hat{\Psi}^\nu$$
$$+ \quad \frac{1}{4}me^{\frac{3}{4}\hat{\phi}}\hat{B}_{pr}\hat{\overline{\Psi}}^\mu \Gamma_{[\mu}\Gamma^{pr}\Gamma_{\nu]}\Gamma_{11}\hat{\Psi}^\nu$$
$$\left. - \quad \frac{1}{2}me^{\frac{5}{4}\hat{\phi}}\hat{\overline{\Psi}}_\mu \Gamma^{\mu\nu}\hat{\Psi}_\nu \right] . \tag{3.43}$$

Using the ansatz (3.42), the definitions of J and Ω and the relations in section 2 as well as the discussion in section 3.5 we can derive the resulting four-dimensional masses. After performing the appropriate rescalings as in section 3.3 the mass terms can be written as

$$S_{\text{mass}}^4 = \int \sqrt{-g} d^4x \left[S^\alpha{}_\beta \overline{\psi_{\mu\alpha}}\gamma^{\mu\nu}\psi_\nu^\beta + (S_\alpha{}^\beta)^* \overline{\psi_\mu^\alpha}\gamma^{\mu\nu}\psi_{\nu\beta} \right] , \tag{3.44}$$

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where $\alpha, \beta = 1, 2$ label the gravitini. The mass matrix S is given by

$$S = \begin{pmatrix} M_1 & D \\ D & M_2 \end{pmatrix} \tag{3.45}$$

with terms defined as below

$$
\begin{aligned}
M_1 &:= \frac{-i}{8} e^{2\phi} \mathcal{V}^{-\frac{1}{2}} \left[\lambda + \int_Y \left(d\mathring{C} \wedge \mathring{B} + \frac{1}{3} m \mathring{B} \wedge \mathring{B} \wedge \mathring{B} \right) + \int_Y (dT + H_3) \wedge U \right. \\
&\qquad \left. + \int_Y \left(\frac{1}{3} mT \wedge T \wedge T + m\mathring{B} \wedge T \wedge T + G_4 \wedge T \right) \right] \\
M_2 &:= -M_1|_{U \to \overline{U}} \\
D &:= \frac{-i}{8} e^{2\phi} \mathcal{V}^{-\frac{1}{2}} \int_Y (dT + H_3) \wedge (i\mathcal{V}^{-\frac{1}{2}} e^{-\phi} \Omega^+) \\
T &:= b - iJ \\
U &:= c + i\mathcal{V}^{-\frac{1}{2}} e^{-\phi} \Omega^- = c + i\sqrt{8} \mathcal{V}^{-\frac{1}{2}} ||\Omega^{cs}||^{-1} e^{-\phi} \Omega^{cs-} \ ,
\end{aligned}
\tag{3.46}
$$

where Ω^+ and Ω^- are the real and imaginary parts of Ω respectively. The four dimensional effective theory will be an $\mathcal{N} = 2$ gauged supergravity. Taking the general form for a gauged supergravity found in [32] we see that using their conventions the gravitino mass matrix is given by

$$S_{\alpha\beta} = \frac{i}{2} P_A^x \sigma_{\alpha\beta}^x L^A \ , \tag{3.47}$$

where the P_A^x are prepotentials, $\sigma_{\alpha\beta}^x$ are Pauli matrices and $L^A := e^{\frac{1}{2} K^{cs}} Z^A$, where K^{cs}, Z^A are defined in section 3.6. Comparing (3.47) with (3.46) we can completely determine the Kähler potential of the vector multiplet sector and the prepotentials of the hypermultiplet sector. We will not go on to do this because in the next section we will see that quite generally this theory will not preserve $\mathcal{N} = 2$ supersymmetry in the vacuum and we will instead have to consider specifying an $\mathcal{N} = 1$ effective theory.

4   Breaking to $\mathcal{N} = 1$

In this section we will explore the implications of the form of the gravitino mass matrix found in the previous section. In order to do this we will specialise to the case where the internal manifold is a particular class of half-flat manifolds. To motivate this choice we will review the most general supergravity solution of massive type IIA on manifolds with $SU(3)$ structure that preserves some supersymmetry constructed in [16]. We will then go on to show that for that class of manifold the low energy theory will not preserve $\mathcal{N} = 2$ supersymmetry in the vacuum and in fact will exhibit spontaneous partial supersymmetry breaking to $\mathcal{N} = 1$. In section 4.3 we will derive the effective action of the resulting $\mathcal{N} = 1$ theory.

4.1   Ten-dimensional massive IIA solutions

In general, the reduction of type-II supergravities on spaces of $SU(3)$ structure should yield an $\mathcal{N} = 2$ supergravity. There are, however, solutions to (massive) IIA supergravity on manifolds of $SU(3)$ structure that preserve supercharges consistent with $\mathcal{N} = 1$ supersymmetry in four dimensions. There were first considered in [13–15], and later generalised in [16]; we shall therefore refer to them

as BCLT (Behrndt-Cvetic-Lust-Tsimpis) solutions. We present here a brief summary of the more general solution in ten-dimensional language.

The metric takes the form of (3.4), with Δ constant, while the fluxes and form fields for the solution take the values

$$
\begin{aligned}
m\hat{B}_2 &= \frac{1}{18}fe^{-\hat{\phi}/2}J + m\widetilde{B} \\
\hat{F}_3 &= \frac{4}{5}me^{7\hat{\phi}/4}\Omega^+ \\
\hat{F}_4 &= f\star\mathbb{1}_4 + \frac{3}{5}me^{\hat{\phi}}J\wedge J \,,
\end{aligned}
\tag{4.1}
$$

where f and $\hat{\phi}$ are constant. $\widetilde{B}$ encodes the non-singlet part of $\hat{B}_2$ and so obeys $\widetilde{B}\wedge J\wedge J = 0$, but is otherwise quite general. A key feature of the solution is that all torsion classes of the compact space vanish except for

$$
\begin{aligned}
\mathcal{W}_1 &= -i\frac{4}{9}fe^{\hat{\phi}/4} \\
\mathcal{W}_2 &= -2ime^{3\hat{\phi}/4}\widetilde{B} \,.
\end{aligned}
\tag{4.2}
$$

Manifolds specified by the torsion classes (4.2) are half-flat, and will play an important role in upcoming sections where we will restrict the internal manifold to lie in this class. We note here that we can always use this type of 'internal' information from a solution in constructing four-dimensional effective actions.

It is informative to see how the fluxes arise in this solution. Considering the torsion classes (4.2) and the relation (3.14) and comparing the fluxes (3.20) and (3.21) with (4.1), we see that the solution corresponds precisely to the case where the fluxes arise purely from the scalar vevs. This will be an important observation later on when we consider what types of fluxes break supersymmetry. A further result of the solution that we shall make use of is that

$$
M_{3/2} = \Delta\left(\frac{\alpha}{|\alpha|}\right)^{-2}\left[-\frac{1}{5}me^{5\hat{\phi}/4} + \frac{i}{6}fe^{\hat{\phi}/4}\right] \,,
\tag{4.3}
$$

where $M_{3/2}$ is the value of the four-dimensional gravitino mass for this solution and α is a constant related to the spinor phase θ that we discussed in section 3.7 and can be consistently set to unity.

4.2 Spontaneous partial supersymmetry breaking

We now want to consider the case where the D terms in the mass matrix vanish. From (3.46) and (4.2) we see that for half flat manifolds $d\Omega^+ = 0$ and so the D terms indeed vanish. The mass matrix diagonalises under this constraint and we see that there appears a mass gap ΔM^2 between the two gravitini given by

$$
\begin{aligned}
\Delta M^2 &= |M_2|^2 - |M_1|^2 \\
&= \frac{1}{32}e^{3\phi}\mathcal{V}^{-1}\left[\int_Y F_3\wedge\Omega^- \int_Y\left(\frac{1}{3}mJ\wedge J\wedge J + F_4\wedge J\right)\right. \\
&\quad \left. + \int_Y dJ\wedge\Omega^- \int_Y\left(\frac{1}{6}fe^{\hat{\phi}/2}J\wedge J\wedge J + mB_2\wedge J\wedge J\right)\right] \,.
\end{aligned}
\tag{4.4}
$$

It is interesting to consider how this mass gap depends on the fluxes. In massless type IIA supergravity such a mass gap requires both RR and NS-NS fluxes to be non-vanishing [33] (despite the

subtleties in doing so, is it possible to see this by taking the limits $dJ, m \to 0$ in (4.4) above). We see that this is not the case here. Either type of flux by itself will generate a mass gap due to a non-vanishing Freud-Rubin parameter [2]. Hence, given general fluxes, the masses of the gravitini are non-degenerate. This implies that we no longer have $\mathcal{N} = 2$ supersymmetry. Indeed such a mass gap corresponds to partial supersymmetry breaking with $\mathcal{N} = 2 \to \mathcal{N} = 1$ for a physically massless lighter gravitino or full supersymmetry breaking with $\mathcal{N} = 2 \to \mathcal{N} = 0$ for a physically massive lighter gravitino

In a Minkowski background, physically massless particles simply have zero mass. In anti-de Sitter (AdS) backgrounds, however, physically massless particles can have non-zero masses [34–36]. This is the case here and so although the masses M_1 and M_2 in (3.46) are non-zero for non vanishing fluxes one of them may still be physically massless. As we saw in section 4.1 fluxes which arise from vevs can preserve $\mathcal{N} = 1$ supersymmetry and therefore have a physically massless gravitino. We can then check that one of our gravitini is indeed physically massless by substituting the solution described in section 4.1 into our mass matrix (3.46) and checking that one of the gravitini has a mass corresponding to the gravitino mass found in the solution.

Putting the solution (4.1) into the gravitino mass matrix and taking care with the rescalings in section 3.3, we find firstly that $D = 0$. This means that $\psi_{1,2}$ are both mass eigenstates, with eigenvalues that obey

$$
\begin{aligned}
M_1 &= \frac{1}{5}me^{5\hat{\phi}/4} - \frac{i}{6}fe^{\hat{\phi}/4} \\
M_2 &= -3M_1 \, .
\end{aligned}
\tag{4.5}
$$

Comparison with (4.3) gives that $|M_1| = |M_{3/2}|$. We therefore see that for the BCLT background, a mass gap opens up for the two gravitini such that the ψ_μ^1 is physically massless and ψ_μ^2 is physically massive. With a slight abuse of terminology we shall therefore refer to the lower mass gravitino as massless and the higher mass one as massive.

For an inexhaustive list of literature discussing partial supersymmetry breaking see [33, 37–42]. Following their discussions we briefly summarise how the matter sector of the theory is affected by the breaking. In the $\mathcal{N} = 2$ theory the fields were grouped into multiplets as described in Table 1. Once supersymmetry is broken these multiplets should split up into $\mathcal{N} = 1$ multiplets. The $\mathcal{N} = 2$ gravitational multiplet will need to split into a $\mathcal{N} = 1$ 'massless' gravitational multiplet and a 'massive' spin-$\frac{3}{2}$ multiplet

$$
\left(g_{\mu\nu}, \psi_1, \psi_2, A^0\right) \to \text{massless } \left(g_{\mu\nu}, \psi_1\right) \ + \ \text{massive } \left(\psi_2, A^0, A^1, \phi_1, \phi_2\right) \, .
\tag{4.6}
$$

Here A^1 is a vector field which has to come from one of the vector multiplets and ϕ_1 and ϕ_2 are spin-$\frac{1}{2}$ Fermions which come from a hypermultiplet. The N_V $\mathcal{N} = 2$ vector multiplets break into n_v massless $\mathcal{N} = 1$ vector multiplets and n_c massless chiral multiplets (with the other fields forming massive multiplets) such that the scalar components of the chiral multiplets span a Kähler manifold $\mathcal{M}^{KV} \subset \mathcal{M}^{SK}$. The N_H $\mathcal{N} = 2$ hypermultiplets break into n_h massless $\mathcal{N} = 1$ chiral multiplets and $N_H - n_h$ massive chiral multiplets with $n_h \leq \frac{1}{2}N_H$. The scalar components of the massless chiral multiplets span a Kähler manifold $\mathcal{M}^{KH} \subset \mathcal{M}^Q$. With mass gaps appearing throughout the matter spectrum we can consider working with an effective $\mathcal{N} = 1$ theory by integrating out the higher physical mass modes. For the case of scalars and fermions this amounts to setting them to zero thereby truncating the matter spectrum of the theory. It is not immediately clear from the

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[2]The case where the Freud-Rubin parameter vanishes will not be a proper supergravity solution and so we do not consider it here.

above considerations exactly which fields to truncate, however we will return to this question in section 4.3 when we construct the $\mathcal{N} = 1$ effective theory.

It is interesting to consider the case where $\mathring{B} = 0 = \mathring{C}$ and the flux arises solely from the vevs of the scalar fields. Then any vacuum of the truncated $\mathcal{N} = 1$ theory where the scalars have non-vanishing vevs for which $\Delta M^2 \neq 0$ will indeed be a valid vacuum of the full $\mathcal{N} = 2$ theory. We will use this observation to find such vacua in section 5.2.

4.3 The $\mathcal{N} = 1$ effective theory

We are interested in constructing the effective $\mathcal{N} = 1$ theory of the physically massless modes. To do this we must explicitly determine how the $\mathcal{N} = 2$ multiplets in Table 1 break into $\mathcal{N} = 1$ superfields and which of these superfields are physically massive or massless. The form of (3.46) suggests that T and U are the correct variables to expand in the chiral superfields. To prove this is the case we will need to show that these superfields span a Kähler manifold with a Kähler potential which matches the one that will be derived from the gravitino mass.

We now turn to the calculation of the $\mathcal{N} = 1$ superpotential and Kähler potential. In the effective $\mathcal{N} = 1$ theory the remaining gravitino mass can be written as

$$M_{3/2} = e^{\frac{1}{2}K} W \, , \tag{4.7}$$

where K is the Kähler potential and W is the superpotential of the theory. It is only this Kähler-invariant combination of W and K that has any physical significance, although it is still natural to decompose (4.7) as

$$e^{\frac{1}{2}K} = \frac{e^{2\phi}}{\sqrt{8}\mathcal{V}^{\frac{1}{2}}} \tag{4.8}$$

$$W = \frac{-i}{\sqrt{8}}\left[\lambda + \int_Y \left(d\mathring{C} \wedge \mathring{B} + \frac{1}{3}m\mathring{B} \wedge \mathring{B} \wedge \mathring{B}\right)\right.$$
$$\left. + \int_Y \left(\frac{1}{3}mT \wedge T \wedge T + m\mathring{B} \wedge T \wedge T + G \wedge T + (dT + H) \wedge U\right)\right] \, . \tag{4.9}$$

This gives a general form for the superpotential and Kähler potential coming from the $\mathcal{N} = 1$ effective action following spontaneous breaking of the $\mathcal{N} = 2$ theory for massive IIA on manifolds of $SU(3)$ structure. The theory will also have D-terms corresponding to the off-diagonal elements of the $\mathcal{N} = 2$ gravitno mass matrix, D in (3.46), which vanish for half-flat manifolds. We will now express W and K in four-dimensional language, assuming that we can expand in the forms of section 3.4 so that

$$T = T^i\omega_i, \qquad U = U^A\alpha_A - \widetilde{U}_A\beta^A \, . \tag{4.10}$$

We can then interpret $T^i, U^A, \widetilde{U}_A$ as the scalar components of chiral superfields, of which the superpotential should be a holomorphic function. Substituting (4.10) into (4.9), we can write the superpotential as

$$W = \frac{-i}{\sqrt{8}}\left[\lambda' + G_iT^i + B_{ij}T^iT^j + k_{ijk}T^iT^jT^k + H_AU^A + \widetilde{H}^A\widetilde{U}_A + (F_i^A\widetilde{U}_A - E_{iA}U^A)T^i\right] \, , \tag{4.11}$$

where $\lambda', G_i, B_{ij}, k_{ijk}, H_A, \widetilde{H}^A$ are four-dimensional constants given by six-dimensional integrals

$$
\begin{array}{rclrcl}
\lambda' & = & \lambda + \int_Y (d\mathring{B} \wedge \mathring{C} + \frac{1}{3}m\mathring{B} \wedge \mathring{B} \wedge \mathring{B}) & k_{ijk} & = & \frac{1}{3}m \int_Y \omega_i \wedge \omega_j \wedge \omega_k \\
B_{ij} & = & m \int_Y \mathring{B} \wedge \omega_i \wedge \omega_j & G_i & = & \int_Y G \wedge \omega_i \\
H_A & = & \int_Y H \wedge \alpha_A & \widetilde{H}^A & = & \int_Y H \wedge \beta^A \, .
\end{array}
\tag{4.12}
$$

As was discussed in section 4.2 turning on fluxes $\mathring{B}, \mathring{C} \neq 0$ will, in general, break supersymmetry further. In the case where supersymmetry is completely broken it does not make sense to talk about superpotentials and superfields. If these fluxes are small relative to the flux originating from the scalar vevs, however, then they can be perturbatively included in the superpotentials (4.9) and (4.11). We therefore display (4.11) as an indication of the class of effective theories that can be obtained from the compactification of massive IIA supergravity on spaces of $SU(3)$ structure. These may be of use in, for example, studying $\frac{1}{2}$-BPS states of such theories as in [43].

To be sure of retaining $\mathcal{N} = 1$ supersymmetry we will only consider fluxes originating from scalar vevs from now on. In that case the superpotential can be written as

$$W = \frac{-i}{\sqrt{8}} \left[\lambda + \int_Y \left(\frac{1}{3} m T \wedge T \wedge T + dT \wedge U \right) \right] . \tag{4.13}$$

Having determined the superpotential of the effective theory we can consider the Kähler potential. To prove that (4.8) is indeed the correct Kähler potential of the truncated theory we need to explicitly perform the truncation and show that the remaining fields form $\mathcal{N} = 1$ superfields, $T^i, U_A, \widetilde{U}_A$, with the corresponding metric. In the Kähler moduli sector it was shown in section 3.5 that indeed the scalars b^i and v^i combine into $T^i = b^i - iv^i$ with Kähler potential (3.34). In the hypermultiplet sector we have N_H hypermultiplets with $4N_H$ real scalar components which are to be truncated to n_h chiral multiplets with $2n_h$ real components. It seems that the correct superfields to form are then

$$U^A = \xi^A + i\sqrt{8}\mathcal{V}^{-\frac{1}{2}} e^{-\phi} \text{Im} \left(||\Omega^{cs}||^{-1} Z^A \right) \tag{4.14}$$

$$\widetilde{U}_A = \widetilde{\xi}_A + i\sqrt{8}\mathcal{V}^{-\frac{1}{2}} e^{-\phi} \text{Im} \left(||\Omega^{cs}||^{-1} F_A \right) . \tag{4.15}$$

Indeed this form for the superfields has been proposed in [44], and also derived in [45] for the case where the partial supersymmetry breaking is induced through an orientifold projection. In our case, however, things are more simple. The internal manifold is a half-flat manifold which has torsion classes

$$\text{Re}(\mathcal{W}_1) = \text{Re}(\mathcal{W}_2) = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0 , \tag{4.16}$$

so the general relations for the proposed Kaluza-Klein basis (3.14) reduce to

$$\begin{aligned} d\omega_i &= E_i \beta_0 \\ d\alpha_0 &= E_i \widetilde{\omega}^i \\ d\widetilde{\omega}^i &= 0 = d\beta^A = d\alpha_{A \neq 0} , \end{aligned} \tag{4.17}$$

for $E_i := E_{0i}$. Applying (4.17) to (2.11) we arrive at

$$dJ = E_i v^i \beta^0 = -\frac{3}{2} \text{Im}(\mathcal{W}_1) \text{Re}(\overline{\Omega}) \tag{4.18}$$

$$d\Omega = Z^0 E_i \widetilde{\omega}^i = i \text{Im}(\mathcal{W}_1) J \wedge J + i \text{Im}(\mathcal{W}_2) \wedge J . \tag{4.19}$$

Equation (4.18) is the motivation behind the statement that the special class of half flat manifolds under consideration do not have any complex structure deformations associated with them. This means that we only have the tensor multiplet and so we only have one chiral superfield left in the truncated theory. This superfield will contain the dilaton ϕ and either ξ^0 or $\widetilde{\xi}_0$. To decide which of the two is to be truncated we can refer to (4.19). We see that for our case, $\text{Re}(\Omega) \propto \beta^0$ and $\text{Im}(\Omega) \propto \alpha_0$. And therefore since only the imaginary part of Ω appears in the effective $\mathcal{N} = 1$

theory we should truncate the field associated with β^0, that is $\widetilde{\xi}_0$. Using the restrictions discussed above we can write the remaining superfield as

$$U_0 = \xi^0 + ie^{-\phi} \left(\frac{-4iZ^0}{F_0} \right)^{\frac{1}{2}}. \tag{4.20}$$

Now inserting (3.18) into (3.1) we get the kinetic term

$$S^U_{kin} = \int \sqrt{-g} d^4x \left[- \left(\frac{F_0}{-4iZ^0} \right) e^{2\phi} \partial_\mu \left(\xi^0 + ie^{-\phi} \left(\frac{-4iZ^0}{F_0} \right)^{\frac{1}{2}} \right) \partial^\mu \left(\xi^0 - ie^{-\phi} \left(\frac{-4iZ^0}{F_0} \right)^{\frac{1}{2}} \right) \right]. \tag{4.21}$$

We see that taking the second derivatives,

$$-\partial_{U^0} \partial_{\bar{U}^0} \ln \left[\frac{e^{4\phi}}{8\mathcal{V}} \right] = \left(\frac{F_0}{-4iZ^0} \right) e^{2\phi}, \tag{4.22}$$

and so (4.8) is indeed the correct Kähler potential and (4.20) is the correct superfield.

5   An example: $SU(3)/U(1) \times U(1)$

Having derived in section 4.3 the form of the $\mathcal{N} = 1$ effective theory on a general manifold with torsion classes (4.16), in this section we will look at an explicit example of such a manifold. Denoting the internal manifold by $\mathcal{Y}$ we will consider the coset space

$$\mathcal{Y} = \frac{SU(3)}{U(1) \times U(1)}. \tag{5.1}$$

In section 5.1 we will derive explicit expressions for J, Ω and the expansion forms on $\mathcal{Y}$. We will then consider the effective theory and derive the superpotential and Kähler potential. Finally we will find supersymmetric minima where all the superfields have non-trivial expectation values.

5.1   Geometry of the coset

In general, a coset manifold $\mathcal{Y} := \mathcal{G}/\mathcal{H}$, where $\mathcal{H} \subset \mathcal{G}$, can be given a non-coordinate basis by taking the generators of $\mathcal{G}$ and removing the generators of $\mathcal{H}$ in a way that is consistent with the embedding of $\mathcal{H}$ in $\mathcal{G}$. We can then construct tensor products of this basis, and it turns out that tensors on the coset are heavily restricted by imposing that they remain invariant under the action of any element of $\mathcal{G}$. This restriction allows us to write the most general $\mathcal{G}$-invariant tensors that can exist on the coset.

The particular case $SU(3)/U(1) \times U(1)$ has been considered in [46], where the two $U(1)$ subgroups are naturally identified with the diagonal Gell-Mann matrices. It was shown that the most general $\mathcal{G}$-invariant two- and three-forms can be written as

$$\begin{aligned} A_{(2)} &= \alpha e^{12} + \beta e^{34} + \gamma e^{56} \\ A_{(3)} &= \delta(e^{136} - e^{145} + e^{235} + e^{246}) + \epsilon(e^{135} + e^{146} - e^{236} + e^{245}), \end{aligned} \tag{5.2}$$

where the $\{e^m\}$ form a basis on the coset space, $\alpha \ldots \epsilon$ are complex coefficients and we define $e^{m_1 \ldots m_p} \equiv e^{m_1} \wedge \ldots \wedge e^{m_p}$. Furthermore, by considering the most general $\mathcal{G}$-invariant symmetric two-tensor on $\mathcal{Y}$, we can define the metric on the coset space to be

$$g_{mn} e^m \otimes e^n := a(e^1 \otimes e^1 + e^2 \otimes e^2) + b(e^3 \otimes e^3 + e^4 \otimes e^4) + c(e^5 \otimes e^5 + e^6 \otimes e^6), \tag{5.3}$$

where a, b, c are real. These three real parameters are the metric moduli of the space $\mathcal{Y}$, and we would like to relate them to the Kähler and Complex Structure forms. Our first step in doing this will be to construct specialisations of the two- and three-forms in (5.2) that obey (2.3), and will therefore be suitable for interpretation as the $SU(3)$-structure forms. Since some of the conditions of (5.2) involve the metric, constructing suitable forms also involves (5.3), and in fact uniquely determines the Kähler and Complex Structure forms in terms of a, b, c. A check on this procedure comes from (2.12). Imposing these constraints the $SU(3)$-structure forms are given by

$$
\begin{aligned}
J &= -ae^{12} + be^{34} - ce^{56} \\
\Omega &= e^{i\varphi}\sqrt{abc}\left[\left(e^{135} + e^{146} - e^{236} + e^{245}\right) - i\left(e^{136} - e^{145} + e^{235} + e^{246}\right)\right] ,
\end{aligned}
\tag{5.4}
$$

where φ is an arbitrary phase which we can set to zero with no loss of generality, a choice that corresponds to choosing the torsion class conventions in (2.11). Now, since the basis on $\mathcal{Y}$ is just a subset of the generators of $\mathcal{G}$, their derivatives will be given in terms of the structure constants for $\mathcal{G}$, and provided the division by $\mathcal{H}$ has been performed adequately these derivatives should remain within $\mathcal{Y}$. Taking derivatives of the forms in (5.4) thus gives—as a specialisation of the result in [46]

$$
\begin{aligned}
dJ &= -(a + b + c)(e^{135} + e^{146} - e^{236} + e^{245}) \\
d\Omega &= 4i\sqrt{abc}(e^{1256} - e^{1234} - e^{3456}) .
\end{aligned}
\tag{5.5}
$$

Comparing (5.5) with (2.11), we see that $\mathcal{Y}$ belongs to the special class of half-flat manifolds defined in (4.16). Having found the appropriate forms and relations for J and Ω we can go on to look for a basis of expansion forms that satisfy (3.13) and (4.17). A consistent set of forms is given by

$$
\omega_1 = -e^{12}, \; \omega_2 = e^{34}, \; \omega_3 = -e^{56}
\tag{5.6}
$$

$$
\widetilde{\omega}^1 = -e^{3456}, \; \widetilde{\omega}^2 = e^{1256}, \; \widetilde{\omega}^3 = -e^{1234}
\tag{5.7}
$$

$$
\alpha_0 = -e^{136} + e^{145} - e^{235} - e^{246}
\tag{5.8}
$$

$$
\beta^0 = -\frac{1}{4}\left(e^{135} + e^{146} - e^{236} + e^{245}\right) .
\tag{5.9}
$$

Note that we have made the choice $E_1 = E_2 = E_3 = 4$, however it would have been possible to choose different values for these parameters had we redefined the forms accordingly, and so this choice is simply for convenience. We have also chosen the normalisation convention $\int_{\mathcal{Y}} e^{123456} = 1$ so that the volume of $\mathcal{Y}$ is given by

$$
\mathcal{V} = abc .
\tag{5.10}
$$

The structure forms J and Ω can be written in terms of this basis as

$$
\begin{aligned}
J &= a\omega_1 + b\omega_2 + c\omega_3 \\
\Omega &= \sqrt{abc}\left(i\alpha_0 - 4\beta^0\right) .
\end{aligned}
\tag{5.11}
$$

It is also worth noting that the torsion classes can be evaluated explicitly in this example, and are given by

$$
\begin{aligned}
\mathcal{W}_1 &= \frac{2i}{3}\frac{a + b + c}{\sqrt{abc}} \\
\mathcal{W}_2 &= \frac{4i}{3}\frac{1}{\sqrt{abc}}\left[a(2a - b - c)e^{12} - b(2b - a - c)e^{34} + c(2c - a - b)e^{56}\right] .
\end{aligned}
\tag{5.12}
$$

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We have therefore been able to derive all the physically relevant quantities in terms of the real metric parameters a, b, c. We can now derive the effective theory arising from a compactification on the space $\mathcal{Y}$.

5.2 The effective theory

In section 5.1 above we showed that the space $\mathcal{Y}$ has three moduli associated with Kähler structure deformations. By comparing (3.33) with (5.11), we are able to relate them to the metric parameters

$$v^1 = a, \; v^2 = b, \; v^3 = c . \tag{5.13}$$

There were no geometric moduli associated with complex structure deformations. In the effective theory we therefore have three superfields T^1, T^2, T^3 from the Kähler structure sector and the superfield U^0 coming from the tensor multiplet. Using the decomposition of Ω^{cs} in (3.29), together with (3.31) and (5.11), gives $F_0 = -4iZ^0$, and so the superfields are

$$
\begin{aligned}
T^i &= b^i - iv^i \\
U^0 &= \xi^0 + ie^{-\phi} .
\end{aligned}
\tag{5.14}
$$

Our knowledge of the coset space also allows us to evaluate the superpotential (4.13) and the Kähler potential (4.8), which become

$$W = \frac{-i}{\sqrt{8}} \left[\lambda + 2mT^1T^2T^3 - 4 \left(T^1 + T^2 + T^3 \right) U^0 \right] \tag{5.15}$$

$$K = -4 \ln \left[-i\frac{1}{2} \left(U^0 - \bar{U}^0 \right) \right] - \ln \left[-i \left(T^1 - \bar{T}^1 \right) \left(T^2 - \bar{T}^2 \right) \left(T^3 - \bar{T}^3 \right) \right] . \tag{5.16}$$

We have now completely specified the $\mathcal{N} = 1$ low energy effective theory on the space $\mathcal{Y}$. It is then natural to ask whether this theory has a stable vacuum. It is a well known result that supersymmetric minima are stable vacua. We therefore look for such a minimum by examining the F-term equations for the superpotential (5.15), which read

$$
\begin{aligned}
D_{T^1}W &= 2mT^2T^3 - 4U^0 - \frac{W}{T^1 - \bar{T}^1} = 0 \\
D_{T^2}W &= 2mT^1T^3 - 4U^0 - \frac{W}{T^2 - \bar{T}^2} = 0 \\
D_{T^3}W &= 2mT^1T^2 - 4U^0 - \frac{W}{T^3 - \bar{T}^3} = 0 \\
D_{U^0}W &= -4 \left(T^1 + T^2 + T^3 \right) - \frac{4W}{U^0 - \bar{U}^0} = 0 ,
\end{aligned}
\tag{5.17}
$$

where the Kähler covariant derivative is given by $D_T := \partial_T + (\partial_T K)$. A solution to these equations can be found by setting $T^1 = T^2 = T^3 =: T$. In this case the equations simplify to the form

$$U^0 = \frac{1}{24T\bar{T}} \left(-T(\lambda + 2m\bar{T}^3) + 3\bar{T}(\lambda + 2mT^3) \right) \tag{5.18}$$

$$0 = -6mT^2\bar{T}^3 - \lambda T\bar{T} - 2m\bar{T}T^4 + 3\lambda T^2 - 2\lambda \bar{T}^2 - 4mT^3\bar{T}^2 + 12mT\bar{T}^4 . \tag{5.19}$$

A physically sensible solution to (5.19) should satisfy $m, e^K, e^{-\phi} > 0$. Imposing these conditions gives a unique solution with $\lambda > 0$ where the vacuum expectation values for the superfield compo-

nents are

$$\langle b^1 \rangle = \langle b^2 \rangle = \langle b^3 \rangle \;=\; -\frac{5^{\frac{2}{3}}}{20}\left(\frac{\lambda}{m}\right)^{\frac{1}{3}}$$

$$\langle v^1 \rangle = \langle v^2 \rangle = \langle v^3 \rangle \;=\; \frac{\sqrt{3}\,5^{\frac{1}{6}}}{4}\left(\frac{\lambda}{m}\right)^{\frac{1}{3}}$$

$$\langle \xi^0 \rangle \;=\; -\frac{5^{\frac{1}{3}}}{20}\left(m\lambda^2\right)^{\frac{1}{3}}$$

$$\left\langle e^{-\phi} \right\rangle \;=\; \frac{\sqrt{3}\,5^{\frac{5}{6}}}{20}\left(m\lambda^2\right)^{\frac{1}{3}} . \tag{5.20}$$

It is easily shown that these values for the scalars satisfy the BCLT equations (4.1). The scalar potential is

$$V = e^K\left[K^{I\overline{J}}D_I W D_{\overline{J}}\overline{W} - 3\,|W|^2\right] , \tag{5.21}$$

where $I, J \ldots = 0,1,2,3$ label the superfields and $K_{I\bar{j}} := \partial_I \partial_{\bar{j}} K$ has inverse $K^{I\bar{J}}$. Substituting (5.20), (5.16) and (5.15) into (5.21) we see that the cosmological constant in the vacuum is given by

$$\langle V \rangle \;=\; -3e^K |W|^2$$

$$=: \;\; \Lambda \simeq \frac{-29.0}{(m\lambda^5)^{\frac{1}{3}}} , \tag{5.22}$$

and so the solution has an anti-de Sitter background. Having found a stable vacuum of the effective $\mathcal{N} = 1$ theory the discussion in section 4.2 further implies that this is also a stable vacuum of the full $\mathcal{N} = 2$ theory. The fact that it is a supersymmetric anti-de Sitter vacuum means that it is stable even if it is a saddle point [34, 35].

The moduli are therefore all stabilised without the use of any non-perturbative effects like instantons and gaugino condensation, or orientifold projections. To our knowledge this is the first example of such a vacuum. Because the stable vacuum arises from vevs of the scalar fields there is no freedom in choosing the flux parameters. The vacuum is in fact determined in terms of only two real parameters λ and m. This sits in contrast with the case of fluxes arising from branes, where the only handle on the generation of flux parameters comes from statistical 'landscape'-type considerations.

We may, however, eventually wish to consider uplifting the vacuum to a Minkowski or a de Sitter vacuum through a mechanism similar to the one used in the KKLT model [19]. Because such a possible uplift will most probably involve non-perturbative effects and new terms in the superpotential it may not leave the form of our solution unchanged. Nevertheless if an uplift leaves the solution unchanged the question of whether it is a full minimum or a saddle becomes important. We will therefore try to answer this question. We can construct a Hermitian block matrix from the second derivatives of the potential with respect to the superfields evaluated at the solution

$$H \;:=\; \begin{pmatrix} V_{I\overline{J}} & V_{IJ} \\ V_{\overline{IJ}} & V_{J\overline{I}} \end{pmatrix} \tag{5.23}$$

$$V_{I\overline{J}} \;=\; e^K K^{L\overline{M}}\partial_L\left(D_I W\right)\partial_{\overline{M}}\left(D_{\overline{J}}\overline{W}\right) - 2e^K K_{I\overline{J}}|W|^2 \tag{5.24}$$

$$V_{IJ} \;=\; -\overline{W}e^K \partial_I\left(D_J W\right) . \tag{5.25}$$

Then for the solution to be a local minimum in all the directions associated with the components of the superfields the matrix H must be positive definite. Inserting the solution (5.20) into (5.25)
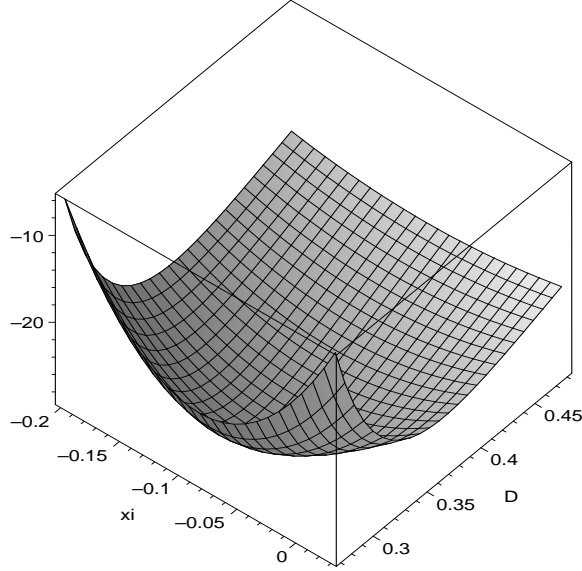
Figure 1: Plot showing the scalar potential for the directions ξ^0 and $e^{-\phi}$ (denoted as D).

we find that out of the eight real eigenvalues only six are positive. This means that there are two real directions for which the potential is at a maximum. We can determine these directions by looking at plots of the potential. Figure 1 shows the scalar potential for the two components of the U^0 (axio-dilaton) superfield at constant T^i with $\lambda = m = 1$. We see that the potential forms a minimum with respect these directions and so the maxima must be in directions associated with the T^i superfields. This raises the possibility that internal spaces with different geometrical structure to $\mathcal{Y}$ may evade this problem. To illustrate this we may consider the potential with the constraint $T^{1,2,3} =: \widetilde{T} =: \widetilde{b} - i\widetilde{v}$ imposed. This would correspond to an internal space with a single Kähler modulus, an example of which might be the coset $G_2/SU(3)$. Figure 2 shows the scalar potential for the directions associated with T at constant U^0. We see that again the potential forms a full minimum. Hence, although this is only an indication of how things might go, it provides motivation for the possibility of other spaces giving full minima and not saddles.

6   Conclusions

In this paper we have shown how the $\mathcal{N} = 2$ four-dimensional effective action for (massive) IIA supergravity on manifolds of $SU(3)$ structure can be constructed from the reduction of fermionic terms. We then went on to show that it is possible to break $\mathcal{N} = 2 \to \mathcal{N} = 1$ spontaneously by having the scalar fields pick up vevs. We derived the most general $\mathcal{N} = 1$ effective theory that can be obtained from such breaking.

Using an example manifold we showed how it is possible to stabilise all the fields in the vacuum without the use of any non-perturbative effects or orientifold projections. This is the first example we are aware of where moduli are stabilised in this manner. The real quantities λ, m are the only free parameters in our solution, which eliminates the need for statistical approaches to parameter space such as the landscape.

The most obvious extension of this work is to look at different explicit examples of half-flat manifolds. In particular other coset manifolds and the Iwasawa manifold. Another open question
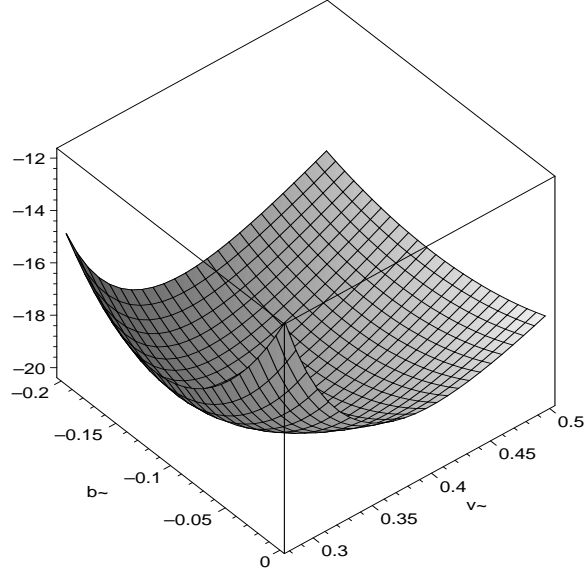
Figure 2: Plot showing the scalar potential for the directions $\widetilde{b}$ and $\widetilde{v}$.

is whether there are any systematic ways to study the moduli spaces of $SU(3)$-structure manifolds that would determine whether our assumptions about the basis for Kaluza-Klein reduction can be proved for the general case.

Although our results depend on some specific features of the massive IIA supergravity, it may be possible to obtain similar $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ spontaneous breaking for other theories, for example IIB and M-theory on manifolds of $SU(3)$ structure or type I and heterotic string theories on manifolds of $SU(2)$ structure. These manifolds offer several globally defined forms in terms of which vev-derived fluxes could be written that might drive the super-Higgs mechanism.

It would also be of interest, having stabilised the moduli, to study the cosmology of the scalars as they roll towards the vacuum. There are also the questions discussed earlier in this paper as to whether the inclusion of non-perturbative effects could lift the vacuum to a de Sitter background.

A further task for looking at phenomenology from these models would be to look at getting a realistic particle content. This could be done either through the use of intersecting branes or through the use of spaces with both the conical singularities needed to obtain chiral fermions, as in [47], and the $A - D - E$ singularities needed to obtain gauge bosons.

Acknowledgments

A   Conventions

Throughout this paper we have used the space-time metric signature $(-, +, +, ...)$. We define the ϵ symbol such that $\hat{\epsilon}_{0123..} := +1$ with $\epsilon := \sqrt{|g|}\hat{\epsilon}$. The indices are raised and lowered with the metric.

The components of a differential p-form ω_p are defined as

$$\omega_p = \frac{1}{p!}\omega_{\mu_1\dots\mu_p}dx^{\mu_1}\wedge\dots\wedge dx^{\mu_p}\ . \tag{A.1}$$

The Hodge star operation $\star$ is defined such that

$$\omega_p\wedge\star\omega_p = \frac{\sqrt{|g|}}{p!}(\omega)_{\mu_1\dots\mu_p}(\omega)^{\mu_1\dots\mu_p}d^Dx\ . \tag{A.2}$$

The contraction of a p-form and a $q\geq p$ form is given by

$$(\omega_p\lrcorner\Omega_q)_{\mu_1\dots\mu_{q-p}} = (\omega_p)^{\nu_1\dots\nu_p}(\Omega_q)_{\nu_1\dots\nu_p\mu_1\dots\mu_{q-p}}\ . \tag{A.3}$$

Dirac matrices anticommute to give

$$\{\Gamma_M,\Gamma_N\} = 2g_{MN}\ . \tag{A.4}$$

Bilinears in spinors are constructed using the operation $\overline{\psi} = \psi^\dagger\Gamma^0$ for Minkowskian signatures and $\overline{\psi} = \psi^\dagger$ for Euclidian signatures, where † denotes Hermitian conjugation.

References

[1] J. Michelson, "Compactifications of type IIB strings to four dimensions with non-trivial classical potential," Nucl. Phys. B **495** (1997) 127 [arXiv:hep-th/9610151].

[2] J. Polchinski and A. Strominger, "New Vacua for Type II String Theory," Phys. Lett. B **388**, 736 (1996) [arXiv:hep-th/9510227].

[3] J. Louis and A. Micu, "Type II theories compactified on Calabi-Yau threefolds in the presence of background fluxes," Nucl. Phys. B **635**, 395 (2002) [arXiv:hep-th/0202168].

[4] G. Dall'Agata, "Type IIB supergravity compactified on a Calabi-Yau manifold with H-fluxes," JHEP **0111**, 005 (2001) [arXiv:hep-th/0107264].

[5] J. P. Gauntlett, D. Martelli, S. Pakis and D. Waldram, "G-structures and wrapped NS5-branes," Commun. Math. Phys. **247**, 421 (2004) [arXiv:hep-th/0205050].

[6] K. Behrndt and C. Jeschek, "Fluxes in M-theory on 7-manifolds: G2, SU(3) and SU(2) structures," arXiv:hep-th/0406138.

[7] K. Behrndt and C. Jeschek, "Superpotentials from flux compactifications of M-theory," Class. Quant. Grav. **21**, S1533 (2004) [arXiv:hep-th/0401019].

[8] T. House and A. Micu, "M-theory compactifications on manifolds with G(2) structure," arXiv:hep-th/0412006.

[9] N. Lambert, "Flux and Freund-Rubin superpotentials in M-theory," arXiv:hep-th/0502200.

[10] A. Lukas and P. M. Saffin, "M-theory compactification, fluxes and AdS(4)," Phys. Rev. D **71**, 046005 (2005) [arXiv:hep-th/0403235].

[11] G. Dall'Agata and N. Prezas, "N = 1 geometries for M-theory and type IIA strings with fluxes," Phys. Rev. D **69** (2004) 066004 [arXiv:hep-th/0311146].

[12] S. Gurrieri, A. Lukas and A. Micu, "Heterotic on half-flat," Phys. Rev. D **70**, 126009 (2004) [arXiv:hep-th/0408121].

[13] K. Behrndt and M. Cvetic, "Supersymmetric intersecting D6-branes and fluxes in massive type IIA string theory," Nucl. Phys. B **676**, 149 (2004) [arXiv:hep-th/0308045].

[14] K. Behrndt and M. Cvetic, "General N = 1 supersymmetric flux vacua of (massive) type IIA string theory," arXiv:hep-th/0403049.

[15] K. Behrndt and M. Cvetic, "General N = 1 supersymmetric fluxes in massive type IIA string theory," Nucl. Phys. B **708**, 45 (2005) [arXiv:hep-th/0407263].

[16] D. Lust and D. Tsimpis, "Supersymmetric AdS(4) compactifications of IIA supergravity," JHEP **0502** (2005) 027 [arXiv:hep-th/0412250].

[17] K. Behrndt, M. Cvetic and P. Gao, "General type IIB fluxes with SU(3) structures," arXiv:hep-th/0502154.

[18] J. P. Gauntlett, D. Martelli, J. Sparks and D. Waldram, "Supersymmetric AdS backgrounds in string and M-theory," arXiv:hep-th/0411194.

[19] S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi, "De Sitter vacua in string theory," Phys. Rev. D **68**, 046005 (2003) [arXiv:hep-th/0301240].

[20] S. Kachru and A. K. Kashani-Poor, "Moduli potentials in type IIA compactifications with RR and NS flux," JHEP **0503**, 066 (2005) [arXiv:hep-th/0411279].

[21] G. Villadoro and F. Zwirner, "N = 1 effective potential from dual type-IIA D6/O6 orientifolds with general fluxes," arXiv:hep-th/0503169.

[22] J. P. Derendinger, C. Kounnas, P. M. Petropoulos and F. Zwirner, "Superpotentials in IIA compactifications with general fluxes," Nucl. Phys. B **715** (2005) 211 [arXiv:hep-th/0411276].

[23] O. DeWolfe, A. Giryavets, S. Kachru, W. Taylor "Type IIA Moduli Stabilization," [arXiv:hep-th/0505160].

[24] E. Bergshoeff, R. Kallosh, T. Ortin, D. Roest and A. Van Proeyen, "New formulations of D = 10 supersymmetry and D8 - O8 domain walls," Class. Quant. Grav. **18**, 3359 (2001) [arXiv:hep-th/0103233].

[25] A. Micu, "Heterotic compactifications and nearly-Kaehler manifolds," Phys. Rev. D **70** (2004) 126002 [arXiv:hep-th/0409008].

[26] L. J. Romans, "Massive N=2a Supergravity In Ten-Dimensions," Phys. Lett. B **169** (1986) 374.

[27] S. Gurrieri, J. Louis, A. Micu and D. Waldram, "Mirror symmetry in generalized Calabi-Yau compactifications," Nucl. Phys. B **654** (2003) 61 [arXiv:hep-th/0211102].

[28] R. D'Auria, S. Ferrara, M. Trigiante and S. Vaula, "Gauging the Heisenberg algebra of special quaternionic manifolds," Phys. Lett. B **610**, 147 (2005) [arXiv:hep-th/0410290].

[29] A. Tomasiello, "Topological mirror symmetry with fluxes," arXiv:hep-th/0502148.

[30] A. Micu, "Background fluxes in type II string compactifications," DESY-THESIS-2003-020 (SPIRES entry)

[31] B. de Wit and A. Van Proeyen, "Potentials And Symmetries Of General Gauged N=2 Supergravity - Yang-Mills Models," Nucl. Phys. B **245** (1984) 89.

[32] L. Andrianopoli, M. Bertolini, A. Ceresole, R. D'Auria, S. Ferrara and P. Fré, "General Matter Coupled N=2 Supergravity," Nucl. Phys. B **476**, 397 (1996) [arXiv:hep-th/9603004].

[33] G. Curio, A. Klemm, D. Lust and S. Theisen, "On the vacuum structure of type II string compactifications on Calabi-Yau spaces with H-fluxes," Nucl. Phys. B **609**, 3 (2001) [arXiv:hep-th/0012213].

[34] P. Breitenlohner and D. Z. Freedman, "Stability In Gauged Extended Supergravity," Annals Phys. **144** (1982) 249.

[35] P. Breitenlohner and D. Z. Freedman, "Positive Energy In Anti-De Sitter Backgrounds And Gauged Extended Supergravity," Phys. Lett. B **115** (1982) 197.

[36] B. E. Gunara, "Spontaneous N=2 → N=1 supersymmetry breaking and the super-Higgs effect in supergravity," (SPIRES entry) and references therein.

[37] S. Cecotti, L. Girardello and M. Porrati, "Two Into One Won't Go," Phys. Lett. B **145**, 61 (1984).

[38] S. Cecotti, L. Girardello and M. Porrati, "An Exceptional N=2 Supergravity With Flat Potential And Partial Superhiggs," Phys. Lett. B **168**, 83 (1986).

[39] S. Ferrara, L. Girardello and M. Porrati, "Spontaneous Breaking of N=2 to N=1 in Rigid and Local Supersymmetric Theories," Phys. Lett. B **376** (1996) 275 [arXiv:hep-th/9512180].

[40] L. Andrianopoli, R. D'Auria and S. Ferrara, "Consistent reduction of N = 2 → N = 1 four dimensional supergravity coupled to matter," Nucl. Phys. B **628**, 387 (2002) [arXiv:hep-th/0112192].

[41] J. Louis, "Aspects of spontaneous N = 2 → N = 1 breaking in supergravity," arXiv:hep-th/0203138.

[42] T. R. Taylor and C. Vafa, "RR flux on Calabi-Yau and partial supersymmetry breaking," Phys. Lett. B **474** (2000) 130 [arXiv:hep-th/9912152].

[43] T. House and A. Lukas, "G(2) domain walls in M-theory," Phys. Rev. D **71**, 046006 (2005) [arXiv:hep-th/0409114].

[44] N. Nekrasov, H. Ooguri and C. Vafa, "S-duality and topological strings," JHEP **0410**, 009 (2004) [arXiv:hep-th/0403167].

[45] T. W. Grimm and J. Louis, "The effective action of type IIA Calabi-Yau orientifolds," arXiv:hep-th/0412277.

[46] F. Mueller-Hoissen and R. Stuckl, "Coset Spaces And Ten-Dimensional Unified Theories," Class. Quant. Grav. **5** (1988) 27.

[47] B. S. Acharya, F. Denef, C. Hofman and N. Lambert, "Freund-Rubin revisited," arXiv:hep-th/0308046.